Canonical Form Exercises
April, 2006

Each of the following matrices has characteristic polynomial $\chi(x) = (x-1)(x-3)^4$. Note that, given this characteristic polynomial, there are three possible Jordan forms, corresponding to the three different partitions of 4. For each matrix A, compute the Jordan form, and find an invertible matrix S in $\mathrm{Mat}_5(\mathbb{Q})$ such that $S^{-1}AS$ is in Jordan canonical form. Also find the elementary divisors, the invariant factors, the minimal polynomial, and the rational canonical form of each matrix.

1.

$$A = \begin{bmatrix} 23 & 18 & -10 & 4 & -8 \\ 10 & 28 & -5 & 2 & -12 \\ 30 & 45 & -12 & 6 & -20 \\ -30 & 18 & 15 & -3 & -8 \\ 20 & 54 & -10 & 4 & -23 \end{bmatrix}$$

It is helpful to know that a basis of the solution space of $(A-3E)v = 0$ is $\left\{ \begin{bmatrix} -1 \\ 0 \\ 0 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \\ 0 \end{bmatrix} \right\}$

and a basis of the solution space of $(A-1E)v = 0$ is $\left\{ \begin{bmatrix} 0 \\ 4 \\ 0 \\ 0 \\ 9 \end{bmatrix} \right\}$.

2.

$$B = \begin{bmatrix} 11 & 1 & -18 & 40 & -4 \\ 20 & 20 & -57 & 124 & -76 \\ -14 & 5 & 30 & -61 & -20 \\ -8 & 2 & 16 & -33 & -8 \\ 4 & 4 & -12 & 26 & -15 \end{bmatrix}$$

It is helpful to know that a basis of the solution space of $(B-3E)v = 0$ is $\left\{ \begin{bmatrix} -1 \\ 0 \\ 4 \\ 2 \\ 0 \end{bmatrix} \right\}$

and a basis of the solution space of $(B-1E)v = 0$ is $\left\{ \begin{bmatrix} 0 \\ 4 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

3.

$$C = \begin{bmatrix} 11 & 2 & -3 & 1 & 0 \\ -8 & 1 & 3 & -1 & 0 \\ 17 & 5 & -3 & 2 & 0 \\ 3 & 3 & 0 & 3 & 0 \\ 2 & 2 & -2 & 2 & 1 \end{bmatrix}$$

It is helpful to know that a basis of the solution space of $(B-3E)v = 0$ is $\left\{ \begin{bmatrix} -1 \\ 1 \\ -2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 3 \\ 3 \\ 0 \end{bmatrix} \right\}$

and a basis of the solution space of $(B-1E)v = 0$ is $\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.