Mathematics 120 Review for Midterm II
November, 2005

The test will be cumulative, but will emphasize more recent material. This review sheet only lists material covered since the first test. Part of the exam will be things you can do by preparing thoroughly from this review sheet. The remainder will be one or more exercises from the homework, or similar to the homework.

Definitions you should know (and you should know examples of each):

1. Direct product of groups, semidirect product of groups
2. Action of a group on a set, orbit, stabilizer, transitive action
3. Centralizer, normalizer.
4. Conjugacy class.
5. Sylow subgroup.

Theorems you should be able to state and prove:

1. Characterization of direct products (Proposition 3.1.5).
2. Characterization of direct products (Proposition 3.1.13).
3. The external semidirect product of groups is a group (Proposition 3.2.3).
4. The primary decomposition theorem for finite abelian groups (i.e. the real guts of the proof in Proposition 3.3.2, not just the one liner in Theorem 3.3.3).
5. Chinese remainder theorem (Theorem 3.3.5).
6. $\mathrm{Aut}(\mathbb{Z}_n) \cong \Phi(n)$ (Proposition 5.3.3).
7. For any action of a finite group, the size of any orbit divides the size of the group. (That's Cor. 5.1.14; include the statement and proof of prop. 5.1.13, but just quote Lagrange.)
8. A group of order p^n has a non-trivial center.
9. Cauchy's theorem (Theorem 5.4.6).

Theorems you should be able to state:

1. The invariant factor decomposition for finite abelian groups (Theorem 3.3.10)
2. The elementary divisor decomposition for finite abelian groups (Theorem 3.3.22).
3. All the Sylow theorems. (You might as well combine the statements into one big statement – I don't care about the numbering of the 3 parts.)

Computations you should be able to do:

1. Compute invariant factor decomposition, elementary divisor decomposition of a finite abelian group already given as a direct product of cyclic groups. Convert between the two canonical decompositions.
2. Compute the element whose existence is claimed in the Chinese remainder theorem.
3. Find all finite abelian groups of a given size.

4. Determine all groups of size pq for a given p and q. E.g. determine all groups of size 2005.

5. Determine the possible number of Sylow subgroups for each prime dividing the order of a finite group. Determine when a certain Sylow subgroup must be normal.

6. It should be second nature that you compute the size of an orbit as $\frac{|G|}{|\mathrm{Stab}(x)|}$. For example number of conjugates of x is $\frac{|G|}{|\mathrm{Cent}(x)|}$, number of conjugates of H is $\frac{|G|}{|N_G(x)|}$