Mathematics 120 Midterm Exam I – F. Goodman
October, 1998
Version 1

Do all problems. Responses will be judged for accuracy, clarity and coherence.

1. Let N be a normal subgroup of a group G. Explain what the quotient group is, and why it is a group. State and prove the homomorphism theorem.

2. Let G be a group in which every non-identy element has order 2. Show that G is abelian.

3. Show that every subgroup and every quotient group of a cyclic group is cyclic.

4. If $\varphi : S_3 \to \mathbb{Z}_3$ is a homomorphism, show that $\varphi(g) = e$ for all $g \in S_3$.

5. Give an example of a group containing elements a and b, each of order 2, such that the product ab has infinite order. Hint: Consider "flips" $J_\theta = R_\theta J R_\theta^{-1}$, where J is the reflection of the $x - y$ plane through the x-axis, and R_θ is the counterclockwise rotation of the $x - y$ plane through angle θ. Recall that J has matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, while R_θ has matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$