YOUR NAME:

YOUR TA'S NAME:

Math 32, SECOND MIDTERM EXAM
APRIL 22, 2003

Scores

1	2	3	4	5	6	total

Instructions:

(1) **Please write your name and the name of your TA prominently at the top of this front page.!**

(2) This exam has 6 questions and 7 pages. The relative weight of the various questions is indicated. Do all the exercises, writing your answers in this exam booklet.

Show your work. Your work will be judged for correctness, completeness, clarity and orderliness. Put your final answer to each question in a box so that it can be located easily.

(3) When you are finished, please turn in your exam paper to your own TA.

(1) (20 points) Consider the function of 3 variables

$$F(x, y, z) = z^2 - x^2 + 3xy,$$

and its level surface

$$z^2 - x^2 + 3xy = 20.$$

The point $A = (1, 4, 3)$ is on this surface.

(a) Compute the gradient of F at $A = (1, 4, 3)$,

$$\nabla F(1, 4, 3).$$

(b) Use the answer to part (a) to write down an equation for the tangent plane to the surface at $A = (1, 4, 3)$.

(2) (20 points) Compute the area integral

$$\int_T xy^2\,dA,$$

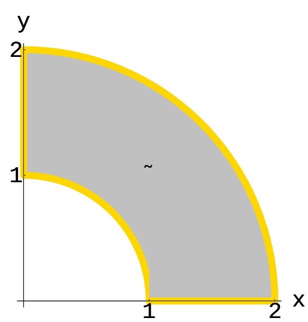
where T is the triangle described by

$$\begin{aligned} &x \geq 0 \\ &y \geq 0 \\ &x/2 + y/3 \leq 1. \end{aligned}$$

(3) (20 points) Compute the area integral

$$\int_E \frac{1}{x^2 + y^2}\, dA,$$

where E is the region indicated in the diagram:



(4) (20 points)

(a) Find the unit tangent vector and parametric equations for the tangent line to the curve

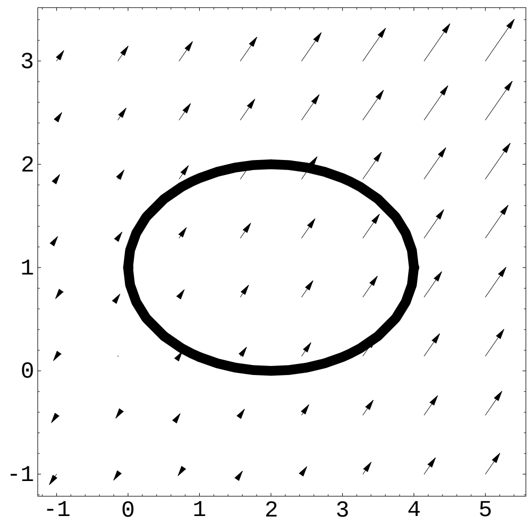
$$X(t) = \begin{bmatrix} t^2 \\ \cos t \\ \sin t \end{bmatrix}$$

at the point where $t = \pi/3$.

(b) With $X(t)$ as above, find the unit tangent vector $T(t)$ at any time t.

(5) (20 points) Consider the vector field $\mathbf{F}(x,y) = \begin{bmatrix} x^2 - y \\ x + y^2 \end{bmatrix}$. Let C be the portion of the parabola $y = x^2 + 1$ traversed from $(0,1)$ to $(1,2)$. Compute the flow $\int_C \mathbf{F}(\mathbf{x}) \cdot d\mathbf{x}$ along the curve C.

(6) (20 points) Let C denote the ellipse $(\frac{x-2}{2})^2 + (y-1)^2$ traversed in the counterclockwise direction. Let D denote the region enclosed by the curve C. Let $\mathbf{F}$ denote the vector field $\mathbf{F}(x,y) = \begin{bmatrix} x+y \\ x+y \end{bmatrix}$. Here is a picture of the curve C, the region D, and the vector field $\mathbf{F}$.



(a) WITHOUT EVALUATING THE INTEGRAL OR PARAMETRIZING THE CURVE, write an expression in terms of x, y, dx, dy for the flow of $\mathbf{F}$ around the curve C, $\int_C \mathbf{F} \cdot d\mathbf{x}$.

(b) Use Green's Theorem to convert the flow integral $\int_C \mathbf{F} \cdot d\mathbf{x}$ to an integral with respect to area over the region D.

(c) Is the vector field $\mathbf{F}$ equal to the gradient of some scalar valued function $\varphi(x,y)$? If so, find φ.