REVIEW FOR SECOND MIDTERM

(1) Remember that all exams are cumulative. I will likely recycle a question from the first exam.

(2) Compute the integral

$$\int_R xy\,dA$$

with respect to area, where R is the triangle bounded by the x–axis, the line $x = 1$, and the line $y = x$.

(3) Consider the integral

$$\int_R xy\,dA$$

with respect to area, where R the quarter disc, $x^2 + y^2 \leq 2$, $x \geq 0$, $y \geq 0$. Convert to an integral in polar coordinates and evaluate.

(4) Sketch the region R over which the integral is performed, giving formulas for the curves forming the boundary of the region:

$$\int_0^3 \Big(\int_{3-y}^{\sqrt{9-y^2}} f(x,y)\,dx\Big)\,dy.$$

Express the integral as an interated integral in the other order.

(5) Find the unit tangent vector and parametric equations for the tangent line to the curve

$$X(t) = \begin{bmatrix} t^2 \\ \cos t \\ \sin t \end{bmatrix}$$

at the point where $t = \pi/3$.

(6) With $X(t)$ as in the previous exercise, find the unit tangent vector $T(t)$ at any time t. Computed $T'(t)$ and show explicitly that it is perpendicular to $T(t)$ for each t.

(7) Find two vectors **a** and **b** such that both are perpendicular to the vector $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, and such that **a** and **b** are perpendicular to each other. Use the vectors **a** and **b** to write parametric equations for a circle of radius 1 centered at the origin and lying in the plane perpendicular to $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

(8) Consider the vector field $\mathbf{F}(x,y) = \begin{bmatrix} x^2 - y \\ x + y^2 \end{bmatrix}$. Let C be the portion of the parabola $y = x^2 + 1$ traversed from $(0,1)$ to $(1,2)$. Compute the flow $\int_C \mathbf{F}(\mathbf{x}) \cdot d\mathbf{x}$ along the curve C Does the value of this integral actually depend on the choice of the curve C or would the flow along any other curve be the same? Replace C with the curve C' consisting of two pieces: the curve $y = 1$ from $(0,1)$ to $(1,1)$, followed by the curve $x = 1$, from $(1,1)$ to $(1,2)$, and compute $\int_{C'} \mathbf{F}(\mathbf{x}) \cdot d\mathbf{x}$. Is the vector field $\mathbf{F}$ the gradient of some scalar function?

(9) With $\mathbf{F}$ and C as in the previous exercise, compute the flow of F across C from left to right.

(10) Evaluate the flow around the curve, $\int_C \mathbf{F}(\mathbf{x}) \cdot d\mathbf{x}$, where C is the unit circle traversed counterclockwise, and $\mathbf{F}(x,y) = \begin{bmatrix} -y \\ x \end{bmatrix}$. Compute the path integral again by using Green's theorem to convert it to an area integral. Is the vector field $\mathbf{F}$ the gradient of some scalar function?

(11) Evaluate the flow out of the curve, 1 $\int_C \mathbf{F}(\mathbf{x}) \cdot \mathbf{n}\, ds$, where C is the unit circle traversed counterclockwise, and $\mathbf{F}(x,y) = \begin{bmatrix} x \\ y^2 \end{bmatrix}$. Compute the path integral again by using Green's theorem to convert it to an area integral.

(12) Study Professor Stroyan's collection of review sheets. I am going to look at them for ideas when I prepare the exam.